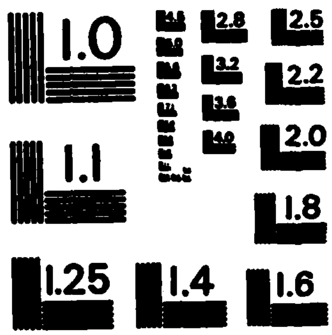
AD-A121 937 SYSTEM OPTIMIZATION BY PERIODIC CONTROL(U) ILLINOIS UNIV AT URBANA DEPT OF MATHEMATICS T E RAGHAVAN JUN 82 AFOSR-TR-82-0850 AFOSR-78-3495 1/1

UNCLASSIFIED F/G 12/1 NL

END
FILMED
DTIC



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

5

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER AFOSR-TR- 82-0850	2. GOVT ACCESSION NO. AD-A121937
3. RECIPIENT'S CATALOG NUMBER	
4. TITLE (and Subtitle) SYSTEM OPTIMIZATION BY PERIODIC CONTROL	5. TYPE OF REPORT & PERIOD COVERED FINAL, 1 Jan 78-31 Dec 81
	6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) T.E.S. Raghavan	8. CONTRACT OR GRANT NUMBER(s) AFOSR-78-3495
9. PERFORMING ORGANIZATION NAME AND ADDRESS Department of Mathematics, University of Illinois at Chicago Circle, Box 4348, Chicago IL 60680	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS PE61102F; 2304/A6
11. CONTROLLING OFFICE NAME AND ADDRESS Directorate of Mathematical & Information Sciences Air Force Office of Scientific Research Bolling AFB DC 20332	12. REPORT DATE June 1982
	13. NUMBER OF PAGES 6
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)	15. SECURITY CLASS. (of this report) UNCLASSIFIED
	15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

DTIC ELECTE DEC 0 9 1982 E

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

In the research project on the theory and applications of Stochastic and Differential Games under the grant, the principal investigator, Professor Raghavan and his former student Professor J. Filar (currently at the Johns Hopkins University) and Professor T. Parthasarathy (currently at the Indian Statistical Institute) considered several problems in the area of Stochastic Games and Differential Games. Most notably they mention a few. This report is a summary of the results of the research.

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

AFOSR-TR- 82-0850

System Optimization by Periodic Control

Final Report on the project AFOSR 78-3495

In the research project on the theory and applications of Stochastic and Differential Games under the grant AFOSR 78-3495 & 78-3495B the principal investigator Professor T.E.S. Raghavan and his former student Professor J. Filar (currently at the Johns Hopkins University) and Professor T. Parthasarathy (currently at the Indian Statistical Institute) considered several problems in the area of Stochastic Games and Differential Games. Most notably we mention a few.

Problem 1 AFOSR 78-3495(1978-79): Let players I and II alternate among finitely many matrix games with the law of motion (namely which matrix game to be played next time knowning the current choices of the players and the current matrix played) completely determined by the current matrix and the choice of one player say player II (the minimizer). If payoff is accumulated with a discount rate or if the payoff is the long run limit per play, can we assert orderfield property? Say, if we know that the data of the game has rational entries can we assert the value and some good stationary strategies have rational components?

This is asserted affirmatively in [1].

Problem 2: Can the result of Problem 1 be extended to non-cooperative bimatrix Stochastic games for stationary Nash equilibria?

This is settled affirmatively in [1].

Under the guidance of Professor T.E.S. Raghavan, J.A. Filar who was supported by the grant as a research assistant during this period submitted his Ph.D. Thesis [4] in the same topic. One of the central questions in the proposal was the following.

Problem 3: For any general Stochastic Game does there exist a value when the payoff is taken to be long run average income per play.

Several people were attacking the same problem all around the world and in

his Ph.D. Thesis Filar obtained the following partial solution.

Theorem: If the decision to terminate is in the hands of one player, say the maximizer then the deciding player has an optimal behavior strategy. The other player has an optimal stationary strategy for cyclic games [6].

From the point of view of applications, stochastic games need to be solved efficiently. During the proposal period 1979-80 the main thrust was in finding finite step algorithms for the Cesaro average payoffs when the law of motion is completely controlled by one player. The main problem in stochastic games in the proposal during (1979-81) was

Problem 4: Find an efficient simplex-like finite step algorithm to find the value of Stochastic Games controlled by one player. Find an algorithm to locate at least one optimal stationary strategy for the two players.

This was settled in the paper [3]. During the special one week session for Game Theory held at the Oberwolfach Mathematics Institute, West Germany, Professor Raghavan was invited to present these findings.

In his Ph.D. Thesis, Filar using the results of [1] solved affirmatively the following problem.

Problem 5: If only one player controls the law of motion but could be different different states does there exist value in stationary strategies. Does the order-field property hold good?

The results have appeared in [5].

Unexpectedly while trying to solve problem 4 several new results that are somewhat curious were obtained [3]. As a sample we mention two theorems.

Theorem: Let the finitely many pure stationary strategies for player I and player II be numbered as 1,2,...m and 1,2,...n. Let $\phi_{ij}(s)$ be the expected Cesaro average income if pure stationary strategy i is used by I and strategy j is used by

II, when the game starts at s. Let v(s) = value of the matrix game $(\phi_{ij}(s))$. If only the same player controls the law of motion at each state, then v(s) coincides with the value of the stochastic game.

Remark: Such a theorem fails to hold in general if the controlling player can be different for different states.

Theorem: The matrix games $(\phi_{ij}(s))$ all have a common optimal strategy for the non-controlling player. This strategy can be used to construct a good stationary strategy for the stochastic game. During the period 1981-82, the main problem was the followin

Problem 6: Find a finite step algorithm for stochastic games where the controller can be different for different states. The lack of any connection to both dynamic programming and matrix games was the key difficulty in this case. While visiting the Game Theory Center at the Katholic University at the Netherlands this problem was mentioned by the principal investigator to Professor Tijs and his student Mr. O.J. Vrieze. The successful finite step procedure based on the joint research work with the Dutch Game Theorists is reported in [7].

Meanwhile these results have been drawing the attention of Game Theoretic researchers all over the world. The algorithms proposed in [1] and [3] are further sharpened in the papers [8], [9]. [10] and [11] for the non-zero sum case the early development of these ideas have triggered interest in [12].

All along, the main thrust was in the solution of stochastic games, even though there were some problems solved in Differential Games as part of the proposal. The main reason is that without understanding the discrete case of multimove games it is not possible to attack continuous versions. However, a specific differential game based on the toy "Etch and Sketch was proposed as a model of continuous conflict. A complete solution to this problem in the spirit of Friedman and Fleming was furnished in an interim report submitted to the AFOSR. Since Differential Games are essentially continuous versions of stochastic games, we concentrated

more on the discrete version of them to avoid technical difficulties of measure theory. The following problem was raised in the proposal during 81-82.

Problem 7: Given a differential game with the dynamics of motion controlled by only one player's control, but the payoff is determined jointly by both players and given that the non-controlling player has only finitely many controls to choose from does there exist a value in relaxed controls. If so, how to find the value.

This problem under certain regularity conditions is solved in [2]. The problems mentioned above and their solutons are reported in the following papers.

[1] T.E.S. Raghavan (with T. Parthasarathy), An orderfield property for Stochastic Games when one player controls transition probabilities, J. Optimization Theory & Applications, vol. 33 No. 3, 1981, 375-392.

[2] T.E.S. Ragahvan (with J.A. Filar), An algorithm for solving S-games and Differential S-games - Accepted for publication - To appear in Siam J. Control and Optimization.

[3] T.E.S. Raghavan (with J.A. Filar) An algorithm for solving undiscounted stochastic games in which one player contrls transitions - Invited address at the Game Theory Session - Oberwolfach Mathematics Institute, West Germany (1980).

[4] J.A. Filar, Algorithms for solving some undiscounted stochastic games. Ph.D. Thesis submitted to the University of Illinois at Chicago Circle, (1979).

[5] J.A. Filar, Orderfield property for stochastic games when the player who controls transitions changes from state to state, J. Optimization Theory and Applications, vol. 34, No. 4, 1981, 503-515.

[6] J.A. Filar, A single loop stochastic game which one player can terminate, Opsearch, vol. 18, 4, 1981, 185-203.

[7] T.E.S. Raghavan (with O.J. Vrieze, S.H. Tijs and J.A. Filar), A finite algorithm for switching control stochastic game, Report 8130, Dec. 1981, Mathematics Institute, Katholic University, The Netherlands.

The researches of the proposal triggered international interest in the problems of our proposal and the following contributions in Stochastic Games resulted.

[8] L.C.M. Kallenberg, Linear Programming and Markovian Decision Problems - Ph.D. Thesis, University of Leiden, The Netherlands (1980).

[9] O.J. Vrieze, Linear programming and undiscounted stochastic games in which one player controls transitions, OR Spectrum, Vol 3, (1981), 29-35.

[10] A. Hordijk & L.C.M. Kallenberg, Linear Programming and Markov Games, 1, Report No. 81-04, University of Leiden 1981.

[11] A. Hordijk & L.C.M. Kallenberg, Linear Programming and Markov Games 2, Report No. 81-06.

[12] V.G. Rothblum, Solving stopping stochastic games by maximizing a linear function subject to quadratic constraints, Game Theory and related topics, O. Moeschlin & D. Pallaschke (editors), North Holland, Amsterdam (1979), 103-105.

The research assistantship under the grant was fruitfully used to help two graduate students for summer supports. While helping in the project the following Ph.D. Thesis was written by a graduate student.

[13] R. Bapat, On Diagonal Products of Doubly Stochastic Matrices Ph.D. Thesis submitted to the University of Illinois at Chicago (1981).

The significant contribution to this important aspect of multimove games resulted in our Ph.D. student M. J.A. Filar getting an assistant professorship at the John Hopkins University. Mr. Bapat after finishing his thesis joined the

Northern Illinois University at DeKalb.

Professor Tijs from the Netherlands visited our campus to do some collaborative research on switching control stochastic games. After some initial success Professor Raghavan visited the Katholic University, The Netherlands to do joint research with Professor Tijs and his student Mr. Vrieze. Such contacts and successful progress would not have been possible without the support of AFOSR.

In summary the principal investigator has successfully completed the main target of solving for good strategies of stochastic games. The major step is from existence to computation. This is achieved for the important class of games controlled by at most one player at each state. Many counter examples exist for other cases to show that a general game is not solvable in stationary strategies.

END

FILMED

1-83

DTIC